

ELECTRICITY CONSUMPTION FORECASTING IN BRAZIL: A COMPARISON AMONG TEMPORAL AND SPATIO-TEMPORAL MULTIVARIATE MODELS

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Resumo: O presente artigo ajustou três modelos multivariados para prever a demanda regional de eletricidade e estimar as respectivas elasticidades de preço, renda, unidades conectadas e temperatura. O Durbin espacial (SDM) dinâmico foi o modelo que apresentou melhor acurácia de previsão. Considerando os resultados do modelo SDM dinâmico, foi confirmada a necessidade de modelar a dependência espacial existente no contexto multivariado. Com relação às elasticidades estimadas, verificou-se persistente inércia temporal, pouca sensibilidade da demanda em relação ao preço e à renda e um expressivo impacto sobre o consumo residencial em decorrência do maior número de unidades conectadas à rede. Além disso, o aquecimento global será responsável pelo aumento da demanda de eletricidade entre 0,90% e 1,49% nos domicílios brasileiros. Novamente, a presença de spillovers espaciais confirmam a existência de interação entre as regiões vizinhas resultando em padrões aglomerativos de consumo de eletricidade no setor residencial.

Palavras-Chave: Demanda regional de eletricidade; Previsão de demanda; elasticidade; dependência espacial; modelos espaço-temporais.

Abstract: This paper fitted three multivariate models to forecast the regional electricity demand and to estimate the respective elasticities of price, income, connected units and temperature. The dynamic spatial Durbin (SDM) was the model that presented the best forecasting accuracy. Considering the results of the dynamic SDM model, it was confirmed the need to model spatial dependence in the multivariate context. With regard to the estimated elasticities, there was persistent temporal inertia, low sensitivity of demand in relation to price and income and a significant impact on residential consumption due to the greater number of units connected to the grid. In addition, global warming will be responsible for increasing the demand for electricity between 0.90% and 1.49% in Brazilian households. Again, the presence of spatial spillovers confirms the existence of interaction between neighbouring regions resulting in agglomerative patterns of electricity consumption in the residential sector.

Keywords: Regional electricity demand; demand forecasting; elasticity; spatial dependence; spatio-temporal models.

Código JEL: D040; C530; R320

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*I Congress Latin American and Caribbean Regional Science Association International
XV Encontro Nacional da Associação Brasileira de Estudos Regionais e Urbanos*

de 11 a 13 de outubro de 2017 - FEA/USP - São Paulo, SP - Brasil

1. INTRODUCTION

The Brazilian Electricity Sector (BES) is a complex hidro-thermal system with continental dimensions and it can be considered unique in the world. The Brazilian Interconnected Electricity System increases the security of the Brazilian Electricity Sector, since it allows the system to operate with synergy and reliability through regional power exchanges (FRANCISCO, 2012). The current model of Brazilian Electricity Sector aims the affordability of tariffs, security of the system and the universalization of access to the Brazilian population. To achieve these targets, the Brazilian Electricity Sector utilizes both centralized and market mechanisms, in what we call a “hybrid” regulatory approach.

In this approach the Utilities have an important role in ensuring the security of the Brazilian Electricity Sector since the households, its main consumer market, are the second largest electricity consumer in Brazil. This sector demands about 25% of the country’s total electricity supply. The household sector is second only to the industrial sector which corresponds to 37,6% of the electricity supply (EPE, 2016). In the Regulated Contracting Environment (RCE), they must have supply contracts to meet the total demand of their markets or suffer penalties if they do not. Thus, there must be constant equilibrium between electricity supply and demand.

The equilibrium of supply and demand leads to electricity blackouts, which cause both social and economic losses. In this context, forecasting the electricity demand plays an essential role to the planning and operation of the distribution segment and it is crucial to decision-making in the electricity sector. Precise forecasts allow the Utilities to plan their future electricity contracts and expand its outreach in order to guarantee productive, allocative and environmental efficiency.

Given the importance of more accurate forecasts for the electricity sector, a huge variety of statistical methods has been used to forecast demand. The forecast models were initiated by Houthakker (1951) and the regression analysis has been the most popular modelling technique for the forecast of the electricity consumption. The unceasing development and enhancement of the statistical tools are essential to provide more precise demand forecasting techniques. The Utilities currently use several forecasting methods to estimate their electricity demand, as for instance, multiple regression (HOUTHAKKER, 1951; ANDERSON, 1973; ZHOU and TENG, 2013), exponential smoothing (CHRISTIAANSE, 1971; EL-KEIB *et al.*, 1995; INFELD and HILL, 1998), the Autoregressive Integrated Moving Average Model (ARIMA) (ELRAZAZ E MAZI, 1989; JUBERIAS *et al.*, 1999) and the VAR/VEC models (MODIANO, 1984; GARCEZ e GHIRARDI, 2003; LIM *et al.*, 2014). Therefore, finding an adequate model of demand forecasting in the electricity sector is not a simple task (ALMESHAI EI and SOLTAN, 2011).

Given the need of models that provide accurate forecasts of electricity demand and the results found by Cabral *et al.* (2017) it is important to examine the influence of spatial interactions of the Utilities’ electricity demand in the multivariate context as well. In other words, this paper expands the work of Cabral *et al.* (2017) when it compares the electricity demand forecasts accuracy among multivariate models under the panel data models. In order to test the hypothesis that the spatial interactions must be included to forecast electricity demand in the Brazilian Electricity Sector (BES), three multivariate methods were fitted: the dynamic panel, the Durbin spatial dynamic model (SDM) and the spatial lag model and spatial autoregressive error with Autoregressive component (SAC-AR (1)). Besides interfering on the

importance of the spatial interactions to electricity demand forecasting, it is also possible to estimate the price and income elasticities, the number of served residences and the temperature through the referred models. According to Baltagi (2008), the literature about forecasting is vast in time series, but yet incipient in panel data.

This paper contributes to the electricity demand modelling and forecasting literature in at least five aspects. The first is that the comparison of the forecast model of multivariate spatial and non-spatial models was never before put in practice in Brazil and it is internationally incipient; the second is that estimating demand elasticities is important so the Utilities segment can anticipate variations of electricity demand and then improving the supply planning. The third is that we presented a theoretical electricity model that considers possible spatial autocorrelation existing in the BES. Fourthly, this empirical analysis provides an alternative tool to the Brazilian Electricity Regulatory Agency (ANEEL), the Energy Research Company (EPE) and to the Electric National System Operator (ONS). At last, the results of the empirical analysis suggest as well that there are possibilities for new research in other areas of the energy sector in which more accurate forecast are important, such as in stream-flow and solar radiation estimates.

This paper is organized in three sections, besides this Introduction. The second section presents the empirical method through the development of a theoretical model of electricity demand for Utilities capable of modelling the spatial interaction existing in the BES. In this section, a methodology and a database are also described. The third section discusses the results found, while in the fourth section the final considerations are refined.

2. EMPIRICAL ESTRATEGY

2.1. Theoretical Model of Electricity Demand Forecasting in the scope of Brazilian Utilities

The analysis of the electricity consumption modelling was inaugurated by the seminal paper of Houthakker (1951). Since then, many articles have contributed to the literature on electricity demand forecasting (WILSON, 1971; TAYLOR, 1975; MODIANO, 1984; IRFFI *et al.*, 2009; ZHOU and TENG, 2013; LIM *et al.*, 2014; CHO *et al.*, 2015). In order to contribute to this literature, it is possible to derive a microeconomic model of electricity demand for Brazilian Utilities.

It is known that the Utilities operate in the “Regulated Contracting Environment” (RCE) of the BES. In this market, consumers are “captive”, meaning consumers can only buy electricity from the Utilities that own the supply concession in their region. Therefore, the RCE is composed of Utilities and their “captive” consumers, for whom the transactions follow the agreements established by ANEEL (SOUZA and LEGEY, 2010).

As mentioned, Utilities must have electricity contracts that fully satisfy their markets. If Utilities contract electricity above 3% or below their observed demand, they are penalized and, as a result, their operating costs increase. Therefore, demand forecasts play an essential role in planning and operation, and are the basis for the decision-making of the Utilities (GARCÍA-ASCANIO and MATÉ, 2010).

Based on the above considerations, it is fundamental for the Utilities to specify adequately the electricity demand model. With a well-specified demand model, Utilities are able to make accurate forecasts to ensure productive, allocative and environmental efficiencies. Based on Houthakker and Taylor (1970), Wilson (1971), Houthakker *et al.* (1974), Houthakker

(1980), Modiano (1984), Amarawickrama and Hunt (2008) and Arisoy e Ozturk (2014), one can specify an individual electric power demand function (D_t) for the Brazilian Utilities as follows:

$$D_t = f(D_{t-1}, P_t, I_t, U_t, T_t) \quad (2.1)$$

Where D_{t-1} is the electricity demand from the previous period; P is the price of electricity; I denotes the average income of consumers; U represents the number of households connected to the grid and T is the temperature. The subscript t indicates time. It is worth mentioning that the price of a possible substitute for electricity, such as natural gas or LPG, was not included in Equation (2.1), since a possible substitution would bring about the need to exchange equipment³.

Assuming D_t as a multiplicative function, Equation (2.1) can be rewritten as follows:

$$D_t = f(D_{t-1}, P_t, I_t, U_t, T_t) = AD_{t-1}^\phi P_t^{\beta_1} I_t^{\beta_2} U_t^{\beta_3} T_t^{\beta_4} \quad (2.2)$$

Where $(\phi + \beta_1 + \beta_2 + \beta_3 + \beta_4) < 1$. Thus, D_t presents decreasing returns of scale.

In order to estimate and forecast the electricity demand of the Utilities, an increasing monotonic transformation of the log-log type was performed in Equation (2.2). After some algebraic rearrangements, we have:

$$\ln(D_t) = \mu + \phi \ln(D_{t-1}) + \beta_1 \ln(P_t) + \beta_2 \ln(I_t) + \beta_3 \ln(U_t) + \beta_4 \ln(T_t) + \varepsilon_t \quad (2.3)$$

Where $\mu = \ln(A)$ is a constant term.

Equation (2.3) can be understood as the equation of electricity demand to be estimated in the scope of the Utilities. Because the functional form specified in Equation (5.3) is log-log, the estimated coefficients can be interpreted as elasticities and represent the direct effects of a percentage variation on a given explanatory variable on the percentage change in electricity consumption. Therefore, β_1 , β_2 , β_3 and β_4 refer to the price and income, units connected to the grid and temperature elasticities, respectively. A negative sign is expected for the price elasticity of demand (β_1) and a positive sign for the coefficients related to the income elasticity (β_2), elasticity referring to the units connected to the grid (β_3) and temperature elasticity (β_4). In addition, ϕ is expected to be positive.

It is worth noting that the models used by the Utilities have an error (ε) in their forecasts of electricity demand. Forecast errors can differentiate one cost-effective model from another that is not. These forecast errors are usually measured through the Mean Absolute Percentage Error (MAPE).

To avoid payment of fines resulting from forecast errors, Utilities seek to benefit from regional exchanges of electricity in their energy purchase contracts. It is worth pointing out that Cabral *et al.* (2017) confirmed that regional electricity consumption in Brazil is spatially

³ In this paper, two goods are said substitutes when the variation of the price of one good generates proportional variation in the demand of the other good, without resulting in exchange of equipment or technology to establish the substitution of one good for the other. In the current case, electricity and natural gas / LPG were not considered as substitutes since, in case of an increase in the price of electricity, for example, the increase in the demand for natural gas / LPG would only be possible through the exchange of technology. In the short term, this technology exchange may not be economically viable for the consumer.

dependent and showed that in the univariate context, modelling spatial interactions increases the forecast performance of the electricity demand forecast models.

Given the above, the existing spatial dependence should be modelled on the Utilities demand function. This dependence can be modelled through the spatial lag of the electricity demand ($\mathbf{W} \ln(D_t)$) and/or spatial spillovers ($\mathbf{W} \ln(\mathbf{X}_t)$ where $\mathbf{X} = \mathbf{W} \ln(P_t), \mathbf{W} \ln(I_t), \mathbf{W} \ln(U_t)$ e $\mathbf{W} \ln(T_t)$) and/or through random shocks of neighbouring regions ($\mathbf{W} \varepsilon_t$). For that matter, the general demand function capable of modelling spatial effects on electricity demand can be specified as follows:

$$\begin{aligned} \ln(D_t) &= \mu + \phi \ln(D_{t-1}) + \rho \mathbf{W} \ln(D_t) + \beta_1 \ln(P_t) + \beta_2 \ln(I_t) + \beta_3 \ln(U_t) + \\ &\beta_4 \ln(T_t) + \Psi_1 \mathbf{W} \ln(P_t) + \Psi_2 \mathbf{W} \ln(I_t) + \Psi_3 \mathbf{W} \ln(U_t) + \Psi_4 \mathbf{W} \ln(T_t) + \varepsilon_t \\ \varepsilon_t &= \lambda \mathbf{W} \varepsilon_t + v_t \end{aligned} \quad (2.4)$$

Where ρ is the spatial autoregressive coefficient that captures the importance of existing spatial interactions (electricity regional exchanges) in the electricity demand of the Utilities. \mathbf{W} is a spatial weights matrix. Ψ measures the importance of exogenous spatial interactions. λ is the spatial autoregressive error parameter. The remainder of the notation remains the same as Equation (2.3).

In order to plan the operation and expansion of the National Interconnected System (SIN), it is possible to propose an aggregate model capable of modelling the electricity demand for the 62 electric power Utilities operating in the BES, based on the individual electricity demand model specified in Equation (2.3). With the sum we have:

$$\begin{aligned} \sum_{n=1}^N \ln(D_t) = \ln(D_{gt}) &= \mu_g + \phi \ln(D_{gt-1}) + \beta_1 \ln(P_{gt}) + \beta_2 \ln(I_{gt}) + \\ &\beta_3 \ln(U_{gt}) + \beta_4 \ln(T_{gt}) + \varepsilon_{gt} \end{aligned} \quad (2.5)$$

Where $g = (1, \dots, G)$ refers to the Distributors and D_{gt} denotes the electricity demand from the Distributor g over time t . Equation (2.5) can be understood as the electricity demand under the panel data structure.

Analogously, it is possible to specify an electricity demand function to the Utilities in the existence of spatial dependence.

$$\begin{aligned} \sum_{n=1}^N \ln(D_t) = \ln(D_{gt}) &= \mu_g + \phi \ln(D_{gt-1}) + \rho \mathbf{W} \ln(D_{gt}) + \beta_1 \ln(P_{gt}) + \\ &\beta_2 \ln(I_{gt}) + \beta_3 \ln(U_{gt}) + \beta_4 \ln(T_{gt}) + \\ &\Psi_1 \mathbf{W} \ln(P_{gt}) + \Psi_2 \mathbf{W} \ln(I_{gt}) + \Psi_3 \mathbf{W} \ln(U_{gt}) + \\ &\Psi_4 \mathbf{W} \ln(T_{gt}) + \varepsilon_{gt} \\ \varepsilon_{gt} &= \lambda \mathbf{W} \varepsilon_{gt} + v_{gt} \end{aligned} \quad (2.6)$$

If it is verified that the regional electricity demand is randomly distributed in time and space, Equation (2.5) must be the equation of electricity demand to be estimated by the Utilities.

2.2. Specification of multivariate models for electricity demand forecasting in Brazil

Dynamic data panel models have been extensively studied in recent decades, because in addition to these models capture the dynamics of the variable of interest, they also control

unobservable heterogeneity among the regional units studied. However, when using regional data, the - theoretical and empirical - econometric literature has emphasized the importance of including spatial effects to obtain consistent and efficient estimates in analyses. Therefore, the researchers' recent interest in space-time models is quite understandable.

In this context, in order to identify the demand forecasting model that minimizes the Utilities' operating costs (model that presents lower forecast MAPE), this paper will estimate three models: dynamic data panel, Durbin dynamic spatial model (Dynamic SDM), spatial lag model and autoregressive spatial error with autoregressive component (SAC-AR (1)).

2.2.1. Dynamic Panel

A data panel is a set that includes cross-sectional data over time. As Wooldridge (2002) points out, the primary motivation for data panel use is to mitigate the problem of omitted variables bias. In the data panel structure, the central question is whether the unobserved effects are (or are not) correlated with the explanatory variables or with the error term. Therefore, consider a linear model of data panel represented by:

$$Y_{gt} = \mu_g + \mathbf{X}_{gt}\boldsymbol{\beta} + v_{gt} \quad (2.7)$$

$$v_{gt} \sim N(0, \sigma^2)$$

Where $g = (1, \dots, G)$ are the observed regions over $t = (1, \dots, T)$ periods of time. Y_{nt} is a vector $GT \times 1$ that denotes the dependent variable; $\boldsymbol{\beta}$ is a vector of exogenous coefficients $K \times 1$ associated to a matrix \mathbf{X} of exogenous covariates of dimension $GT \times K$; μ_n is a $GT \times 1$ vector of unobserved, time-constant effects specific to each *cross-sectional* unit (individual effects); v_{gt} is a white noise of $GT \times 1$ dimension.

The unobserved effects bias of the estimates by Ordinary Least Squares (OLS) should be treated using the fixed effects (FE) or random effects (RE) models, according to the Hausman (1978) test. FE is estimated by OLS that uses a within transformation to remove the unobserved effect (μ_g). The estimation by RE is used when the observed effect is not correlated to all the explanatory variables and it involves estimation by generalized least squares (GLS).

However, a limitation of the data panel models is that they do not incorporate the possible temporal dynamics of the dependent variable. Such limitation is overcome by the estimations of the dynamic models with panel data developed from estimates of the Generalized Method of Moments (GMM). The estimation of dynamic panel data by GMM is associated with Arellano and Bond (1991), Arellano and Bover (1995) and Blundell and Bond (1998).

The use of dynamic models for panel data is justified by the fact that many variables of interest relate to one another and to their previous values. In this sense, models that consider the dependent variable to be temporarily out of time as an explanatory variable seem to adhere better to its behaviour over time. In this case, the GMM estimation allows taking into account the persistence of dependent variable over time.

Thus, dynamic relationships can be specified in a data panel model by including a temporarily lagged dependent variable as follows:

$$Y_{gt} = \mu_g + \alpha Y_{gt-1} + \mathbf{X}_{gt}\boldsymbol{\beta} + v_{gt} \quad (2.8)$$

$$v_{gt} \sim N(0, \sigma^2)$$

Where Y_{gt-1} refers to the time lag of the dependent variable and α is the associated coefficient. The rest of the notation remains the same as Equation (2.7).

Considering the probable inertia, as well as the determinants of regional electricity consumption specified in Equation (2.5), the dynamic panel model expressed by Equation (2.7) can be rewritten as follows:

$$\ln(D)_{gt} = \mu_g + \alpha \ln(D)_{gt-1} + \beta_1 \ln(P_{gt}) + \beta_2 \ln(I_{gt}) + \beta_3 \ln(U_{gt}) + \beta_4 \ln(T_{gt}) + v_{gt}$$

$$v_{gt} \sim N(0, \sigma^2) \quad (2.9)$$

In the dynamic panel model, as in the linear data panel model, the unobserved effects inherent to each region (μ_g) will be treated by the fixed effects or random effects method and the choice of method will be made based on the Hausman test (1978). From this, the GMM estimator of Arellano and Bond (1991) equips the explanatory variables ($\Delta X_{gt} = X_{gt} - X_{gt-1}$) that are not strictly exogenous with their available lags. Arellano and Bond (*ibid.*) propose the use of lagged variables in at least two periods ($t - 2$), as an instrument for the first difference equation.

2.2.2. Dynamic Spatial Durbin Model (dynamic SDM)

Regional scientists have shown that spatial dependence on data can alter, and even bias, the results obtained through classical econometric methods. Several researches, such as Rey and Montouri (1999) and Badinger *et al.* (2004), established the importance of integrating spatial and temporal lags in the econometric analysis of regional data. However, the literature on temporal and spatial dynamics has progressed autonomously, with little interaction between these two aspects (Beenstock and Felsenstein, 2007).

Concerning regional electricity consumption in Brazil, it is plausible to state - according to what was discussed in section.2.1 and considering the results of Cabral *et al.* (2017) - that there are spatial spillovers. However, the models that deal with spatial and temporal dynamics concomitantly can be considered exceptions, one of them being the spatial dynamic panel models.

More specifically, Debarsy *et al.* (2012) consider an autoregressive spatial dynamic panel model that allows us to represent spatial dynamics by including contemporary spatial lags of both the dependent variable and the covariates (WY_{gt} e WX_{gt}) while the time dynamics is modelled by an inclusion of the temporarily lagged variable (y_{gt-1}) in the regression. Also, the time lag of the spatially lagged dependent variable (Wy_{gt-1}) treats spatial and temporal dynamics in an overlapping way. Such a model was named by the authors as dynamic Spatial Durbin Model (dynamic SDM) and can be formalized as follows:

$$Y_{gt} = \mu_g + \alpha Y_{gt-1} + \rho_1 WY_{gt} + \rho_2 WY_{gt-1} + X_{gt}\beta + WX_{gt}\psi + v_{gt}$$

$$v_{gt} \sim N(0, \sigma^2) \quad (2.10)$$

In the case of regional electricity demand, the reason for including the spatial lag of the dependent and explanatory variables (WY_{gt} e WX_{gt}) is that the electricity consumption of a region is probably influenced by the average consumption of electricity in neighbouring regions because Brazilian regions are socially and economically interrelated. The inclusion of Wy_{gt-1} in the specification intends to test the hypothesis that there would be an inertia in the electricity demand of the neighbouring regions capable of impacting the observed regional demand.

Thus, Equation (2.8) can be specified taking into account the regional model of electric power demand described in Equation (2.6) as follows:

$$\begin{aligned} \ln(D_{gt}) &= \mu_g + \alpha \ln(D_{gt-1}) + \rho_1 \mathbf{W} \ln(D_{gt}) + \rho_2 \mathbf{W} \ln(D_{gt-1}) + \beta_1 \ln(P_{gt}) + \\ &\beta_2 \ln(I_{gt}) + \beta_3 \ln(U_{gt}) + \beta_4 \ln(T_{gt}) + \Psi_1 \mathbf{W} \ln(P_{gt}) + \Psi_2 \mathbf{W} \ln(I_{gt}) + \\ &\Psi_3 \mathbf{W} \ln(U_{gt}) + \Psi_4 \mathbf{W} \ln(T_{gt}) + v_{gt} \\ v_{gt} &\sim N(0, \sigma^2) \end{aligned} \quad (2.11)$$

The unobserved effects will be treated, once again, through FE or RE models. In order to estimate the parameters of Equation (2.11) efficiently and consistently, the generalized methods of moments (GMM) proposed by Blundell and Bond (1998) will be used. It should be noted that the application of dynamic SDM is pioneer in the electric sector both at national and international level.

2.2.3. Spatial Lag Model and Autoregressive Spatial Error With Autoregressive Components (SAC-AR(1))

Given the importance of spatial interactions for regional electricity consumption and, more specifically, if the patterns of electricity consumption observed in neighbouring regions are correlated with local consumption and the unobserved variables are spatially correlated, a model capable of treating both spatial effects is the SAC model. Such model combines spatial autoregressive and autoregressive errors and thus allows spatial spillovers to be modelled both in the endogenous variable and in the error term.

However, as specified in Equation (2.6), it is plausible to assume that the current electricity consumption has an inertial component with the previous period. Thus, it is necessary to include the temporal autoregressive component in the econometric specification. In order to contemplate the inertial component of electricity consumption, Cho *et al.* (2015) extended the SAC model, calling it the SAC-AR(1) model.

From the definitions made above, the SAC-AR (1) model, in its matrix form, can be specified as follows:

$$\begin{aligned} Y_{gt} &= \mu_g + \alpha Y_{gt-1} + \rho \mathbf{W} Y_{gt} + \mathbf{X}_{gt} \boldsymbol{\beta} + \varepsilon_{gt}; \text{ onde } |\alpha| < 1 \text{ e } |\rho| < 1 \\ \varepsilon_{gt} &= \lambda \mathbf{W} \varepsilon_{gt} + v_{gt}; \text{ onde } |\lambda| < 1 \\ v_{nt} &\sim N(0, \sigma^2) \end{aligned} \quad (2.12)$$

Where ε_{gt} is a vector $GT \times 1$ of spatially lagged errors and λ is the spatial autoregressive coefficient of the residues. The remainder of the notation remains the same as Equation (2.11).

Equation (2.12) can be easily extrapolated to estimate the electricity demand of the “Regional Equivalent Utilities” as follows:

$$\begin{aligned} \ln(D_{gt}) &= \mu_g + \alpha \ln(D_{gt-1}) + \rho_1 \mathbf{W} \ln(D_{gt}) + \beta_1 \ln(P_{gt}) + \beta_2 \ln(I_{gt}) + \\ &\beta_3 \ln(U_{gt}) + \beta_4 \ln(T_{gt}) + \varepsilon_{gt} \\ \varepsilon_{gt} &= \lambda \mathbf{W} \varepsilon_{gt} + v_{gt} \end{aligned} \quad (2.13)$$

As in the other models specified until now, the unobserved heterogeneity (μ_n), inherent to each Utility, will be treated by the FE or RE model. The estimation of SAC-AR (1) for the spatial data panel, specified in Equation (2.13), will be performed using the maximum likelihood (ML) method proposed by Elhorst (2010) and Lee and Yu (2010). The choice of the ML method is due to the fact that the ML method treats the endogeneity of the $W\ln(D_{gt})$ variable in a simple way, besides being a very widespread method in the literature.

2.3. Database

The database used consists of a monthly data panel from January 2004 to December 2014 for the five Brazilian regional units (North, Northeast, Central-West, Southeast and South). Therefore, the panel contains 132 observations for each region totalling 660 observations. The variables used for the estimation of the three proposed models are: regional consumption of electric power; number of residences served in each region; regional average tariff; per capita average regional income of formal activity; and average temperature.

Regarding the data sources, the regional electricity consumption, the number of residences served in each region and the average regional tariff are derived from the Decision Support System (SAD) of the National Agency of Electrical Energy (ANEEL). The per capita regional average income of the formal activity was obtained by the ratio between the monthly regional wage mass and the number of workers in the region. The data of the monthly regional salary were extracted from the General Cadastre for Employed and Unemployed (CAGED). The monthly amount of workers was obtained through the matching of employment data from the Annual Social Information Report (RAIS) and CAGED, both available from the Ministry of Labor (MT).

Regarding the climatic variable, the average temperature of the regions was taken from the Meteorological Database for Teaching and Research of the National Institute of Meteorology (BDMEP / INMET). Table 1 gives a schematic summary of the variables used in the models and their respective sources.

Table 1: Summary of data sets used.

Variable	Description	Unit	Source
D	Regional Electric Power Consumption	GWh	SAD/ANEEL
P	Regional Average Tariff	R\$	SAD/ANEEL
I	Per Capita Regional Average Income	R\$	CAGED/RAIS/MT
U	Number Of Residences Served	Quantity	SAD/ANEEL
T	Regional Average Temperature	°C	BDMEP/INMET

3. RESULTS

Knowing the existence of unobserved effects may lead to biased estimates of regional electricity consumption, the Hausman (1978) test was performed to identify whether such effects would be adequately modelled by fixed effects or random effects. The Hausman test rejected the null hypothesis that random effects would be consistent and thus the fixed effects model is the best choice for the treatment of the unobserved effects on the regional electricity demand in Brazil. Therefore, dynamic panel models, dynamic Durbin and SAC-AR (1) were estimated with correction of the unobserved effects through fixed effects modelling. The results of the models are reported in Table 2.

The presence of the time lag on the right side of the regression of the estimated models operates as an endogenous regressor. Thus, in order to address this temporal endogeneity, the Arellano-Bover (1995) estimator, Blundell and Bond (1998), an Arellano-Bond (1991) estimator extension, was implemented to estimate the dynamic panel model. This estimator consists of the estimation of the first differences in Equation (2.9) by GMM. With this, the method removes the specific unobserved time-invariant specific effects.

However, it is necessary to verify their consistency of the coefficients by the Arellano-Bond autocorrelation test to allow the interpretation of the coefficients. Although the coefficients are reported for the level of the variables, the test is performed on the residuals in difference. The hypotheses tested refer to the absence of serial correlation of first and second order, it being desirable to reject the first and not to reject the second. The consistency of the Arellano-Bond estimator depends, therefore, on the assumption that there is no second-order serial correlation in the residuals in the first difference, $E(\Delta v_{g,t} \Delta v_{g,t-2}) = 0$, which implies the absence of serial autocorrelation in level residuals, as assumed by the method. The values of the Arellano-Bond autocorrelation test for the first and second order were, respectively, -2.138 ** and -0.461. Therefore, it is possible to affirm that the estimates are consistent since it is not possible to reject the null hypothesis of absence of second order serial correlation for the residential demand.

It is also necessary to test if the residuals of the dynamic panel are spatially autocorrelated, considering that in the presence of spatial dependence, the estimates are inefficient if this dependence manifests itself in the form of spatial autoregressive error. Such estimates can also be biased if this dependency takes the form of spatial lag of the dependent variable and/or spatial spillovers of the exogenous variables. Based on the CD-Pesaran test (6.60 ***), there is *cross sectional* spatial dependence between regions. To confirm this spatial dependence, we computed Moran's I test (0.257 ***) - spatial autocorrelation statistic -, which also indicated the existence of spatial dependence on residuals at the 1% level of significance.

Table 1: Estimation results of multivariate models for regional electricity demand

Demanda [$\ln(D_{gt})$]	Dynamic Panel ^a	Dynamic SDM ^b	SAC-AR(1) ^c
Demand _{t-1} [$\hat{\alpha}\ln(D_{gt-1})$]	0.889*** (0.012)	0.713*** (0.023)	0.731*** (0.024)
Tariff [$\hat{\beta}_1\ln(P_{gt})$]	-0.078*** (0.014)	-0.152*** (0.026)	-0.166*** (0.030)
Income [$\hat{\beta}_2\ln(I_{gt})$]	0.030*** (0.005)	0.055*** (0.021)	0.097*** (0.025)
Units [$\hat{\beta}_3\ln(U_{gt})$]	0.106*** (0.013)	0.39*** (0.046)	0.44*** (0.045)
Temperature [$\hat{\beta}_4\ln(T_{gt})$]	0.016** (0.008)	0.037*** (0.010)	0.059*** (0.001)
WTariff [$\hat{\Psi}_1\mathbf{W}\ln(P_{gt})$]		0.058*** (0.015)	
WIncome [$\hat{\Psi}_2\mathbf{W}\ln(I_{gt})$]		-0.024* (0.013)	
WUnits [$\hat{\Psi}_3\mathbf{W}\ln(U_{gt})$]		-0.100*** (0.036)	
WTemperature [$\hat{\Psi}_4\mathbf{W}\ln(T_{gt})$]		0.011 (0.007)	
[$\hat{\rho}_1\mathbf{W}\ln(D_{gt})$]		0.242*** (0.024)	0.014 (0.043)
[$\hat{\rho}_2\mathbf{W}\ln(D_{gt-1})$]		-0.185*** (0.023)	
[$\hat{\lambda}\mathbf{W}\varepsilon_{gt}$]			-0.206*** (0.056)
Constant	0.084** (0.042)	-0.221 (0.295)	—
AIC	-3.497	-3.710	-2.969
BIC	-3.489	-3.693	-2.959
MAPE	0.193	0.147	0.374
Number of observations	660	660	660

Notes: Stand errors in parentheses;

*** p<0.01, ** p<0.05, * p<0.1;

^a results obtained using the *xtdpd*sys comand of STATA 12.

^b results obtained using the *spregd*pd comand of STATA 12.

^c results obtained using the *xsmle* comand of STATA 12.

Thus, the need to model spatial dependence on regional electricity consumption in Brazil is also confirmed in the multivariate context. In order to include existing spatial interactions, the dynamic SDM and SAC-AR (1) models were estimated based on Equations (2.11) and (2.13), respectively. The results of both models are presented in columns three and

four of Table 1 and, because the variables are specified in logarithm, their coefficients directly reflect the demand elasticities.

It is possible, through the results, to attest that the dynamic SDM model has better goodness of fit, since it presented the lowest AIC and BIC information criteria. Another result that confirms the choice of the dynamic SDM model as the most adequate to predict the regional electricity demand in Brazil refers to the forecast accuracy. The predictive performance of the dynamic SDM was superior to the other models because it presented the lowest MAPE. Therefore, considering the spatial interactions improves the prediction accuracy of regional electricity demand in Brazil.

The improvement of forecast accuracy by spatial models was also found by Ohtsuka *et al.* (2010) for Japanese Utilities. In the Brazilian case, Cabral *et al.* (2017) confirmed the predictive performance superiority of the space-time model (ARIMASp) to predict the electricity demand of the “Southeast Equivalent Utility” in the univariate context. Blásquez Gomez *et al.* (2013) for Spanish provinces and Cho *et al.* (2015) for South Korean regions have found evidence that spatial models are more suitable to estimate residential electricity demand.

According to Le Sage and Pace (2009), the total effects in this case are not given by the estimated coefficients. In an SDM model, to have the impact of both direct and indirect effects provided by the spatial interaction between regions of an explanatory variable X_k on the regional electricity consumption in Brazil, it is necessary to multiply the estimated contemporary parameters by spatial transformation $(I_g - \mathbf{W}\rho_1)^{-1}$, where I_g is an identity matrix $G \times G$. Because the spatial weighting matrix \mathbf{W} is row standardized, the spatial multiplier becomes $(I_g - \rho_1)^{-1}$.

Thus, as explained by Elhorst (2010), the interpretation of the results of a dynamic SDM model must be careful, because the specification of this model presents the temporally lagged dependent variable $\ln(D_{gt-1})$, spatially $(\mathbf{W}\ln(D_{gt}))$ and temporal and spatially lagged $\mathbf{W}\ln(D_{gt-1})$, besides the spatially lagged explanatory variables $(\mathbf{W}\ln(X_{gt}))$. Therefore, the interpretation of the total marginal effects of the dynamic SDM model must take into account all these effects. The total marginal effects of the dynamic SDM model proposed by Elhorst (2010) are found from the following transformation of Equation (2.11):

$$\begin{aligned} \ln(D_{gt}) = & (I_g - \rho_1 \mathbf{W})^{-1} (\alpha I_g - \rho_2 \mathbf{W}) \ln(D_{gt-1}) + (I_g - \rho_1 \mathbf{W})^{-1} (\beta_1 + \\ & \Psi_1 \mathbf{W}) \ln(P_{gt}) + (I_g - \rho_1 \mathbf{W})^{-1} (\beta_2 + \Psi_2 \mathbf{W}) \ln(I_{gt}) + (I_g - \\ & \rho_1 \mathbf{W})^{-1} (\beta_3 + \Psi_3 \mathbf{W}) \ln(U_{gt}) + (I_g - \rho_1 \mathbf{W})^{-1} (\beta_4 + \Psi_4 \mathbf{W}) \ln(T_{gt}) + \\ & (I_g - \rho_1 \mathbf{W})^{-1} v_{gt} \end{aligned} \quad (2.14)$$

From this transformation, the empirical results of the dynamic SDM model also allow the decomposition of demand elasticities into direct, indirect and total effects. According to LeSage and Pace (2009), the direct effect is the average of the elements that appear on the main diagonal of $(I_g - \rho_1)^{-1}$, that is, $\bar{E}_{direct} = N_G^{-1} \text{dash} \left[(I_g - \rho_1)^{-1} \right] I_g \pi_k$ while the total effect is the average of the sum of the rows or columns of the matrix $(I_g - \rho_1)^{-1}$, that is, $\bar{E}_{total} = N_G^{-1} (I_g - \rho_1)^{-1} I_g \pi_k$. The indirect effect is obtained by the difference between the total effect and the direct effect ($\bar{E}_{indirect} = \bar{E}_{total} - \bar{E}_{direct}$).

In the studied case, the \bar{E}_{direct} can be understood as a change in given variable in the n^{th} region on the electricity demand of the n^{th} region. However, $\bar{E}_{indirect}$ refers to a change in such variable in neighboring regions on electricity demand of the n^{th} region. The \bar{E}_{total} is a change in such variable in the n^{th} region on the electricity demand of the n^{th} region plus the effect of the neighbouring regions, exerting a feedback effect on the electricity demand in the n^{th} region. Table 2.3 shows the decomposition of the elasticities that were statistically significant.

Table 2: Decomposition of the regional electricity demand elasticities in Brazil estimated using the dynamic SDM model.

	Direct effect	Indirect effect	Total effect
Demand _{t-1} [$\hat{\alpha} \ln(D_{gt-1})$]	0,941	-0,244	0,697
Tariff [$\hat{\beta}_1 \ln(P_{gt})$]	-0,201	0,077	-0,124
Income [$\hat{\beta}_2 \ln(I_{gt})$]	0,073	-0,032	0,041
Units [$\hat{\beta}_3 \ln(U_{gt})$]	0,515	-0,132	0,383
Temperature [$\hat{\beta}_4 \ln(T_{gt})$]	0,049	0,015	0,063

It is interesting to note that implemented policies in such region in order to reduce demand and/or conserve electricity will not only have effects in the region where the policy was implemented but will also have effects in neighbouring regions. The indirect effect is that, for example, policy of tariff increase in 1% in one region will be able to reduce the demand for electricity from neighbouring regions by 0.049%.

In relation to the total effect of the temporal inertia of electricity consumption in the Brazilian regions ($\ln(D_{gt-1})$), its impact is not given by its coefficient of 0.713 but by the value of 0.697. It can be assumed that the temporal inertia of regional consumption is important to explain current consumption. This high value of the consumption coefficient in the previous period reveals that the electricity consumption habits are relatively stable in Brazilian households.

Regarding the price elasticity of demand, the total effect of electricity price on consumption is -0.124. It is therefore expected that 1% increase in the tariff will be followed by a drop of 0.124%, on average, in the quantity demanded, ceteris paribus. The price inelasticity can be attributed to the absence of substitute energy sources and/or alternatives to electricity in Brazilian households. This result suggests that a policy of tariff increase would not be effective as an instrument to reduce electricity consumption and, consequently, would not guarantee improvements in future electricity generation conditions. Given the price inelasticity of demand, it is expected that the System of Tariff Flags, introduced in 2015, will have a reduced impact on the electricity consumption in Brazil.

As to income, an average variation of 0.041% in electricity consumption is expected due to a 1% variation in per capita income. Thus, the regional electricity demand in Brazil is not very sensitive to changes in the income of formal workers. One of the possible explanations for the inelasticity of demand in relation to income can be attributed to the rationing of electric power occurred in 2001 that was able to change the consumption habits of Brazilian consumers.

The inelasticity of electricity demand in Brazil regarding price and income is connected to the values estimated by the national literature, such as the studies of Modiano (1984), Andrade and Lobão (1997), Schdmit and Lima (2004) And Rodrigues et al. (2013).

No studies were found that estimate such elasticity in the national literature, concerning the served units. By analysing the results, it can be seen that the 1% increase in total of new connections to the grid increases the regional electricity consumption by 0.383%. In other words, new connections to the grid have positively impacted regional electricity consumption. This result is interesting, since it allows Utilities to anticipate variations in electricity demand due to access universalization programs - such as the Light for All Program - and electric energy theft reduction programs that encourage the legalization of consumption - Social Tariff Program, for example.

As for temperature, global climate change has caused natural catastrophes and extreme levels - low or high – of temperature. In Brazil, the National Institute for Space Research (INPE) forecasts an increase of 3°C to 5°C in temperature due to global warming. As mentioned, temperature became one of the main drivers of electricity demand. Despite this, few studies have been found that consider the influence of temperature on the electric power demand in Brazil. It can be mentioned the studies of DePaula and Mendelsohn (2010), Hollanda et al. (2012) and Rodrigues et al. (2013) that incorporated the climate as an explanatory factor of residential electricity demand.

In this paper, a positive relation between the demand of electricity and the temperature was found. Thus, a percentage change in the electricity demand of 0.063% is due to the 1% increase in temperature. Otherwise, a variation of 1°C in temperature impacts the electricity demand by 0.299%, which evidences a potential adaptive strategy of the Brazilian regional electricity consumption to the projections of climate change in the country.

Finally, spatial interactions are important factors to explain the regional electricity demand, since the parameters of the consumption ($\hat{\rho}$) spatial lags and the spillovers of the explanatory variables ($\hat{\psi}$) were statistically significant. The positive spillovers in residential consumption found by this study may indicate dependence on socioeconomic activities, lifestyle trends and consumption behaviour among neighbouring regions that result in similar patterns of regional demand for residential electricity (JEENINGA and HUENGES WAJER, 1999; Al., 2015).

4. FINAL CONSIDERATIONS

Given the importance of households for the electricity demand in Brazil, this study fitted three models to forecast the regional electricity demand, as well as to estimate the respective demand elasticities in the monthly period from 2004 to 2014. Through the results, it was confirmed the need to model spatial interactions in the regional electricity consumption, with the dynamic SDM model being the most appropriate. This model presented a better predictive performance with forecast MAPE of 0.147.

From the results of the dynamic SDM, it can be seen that the consumption of electricity in the residential sector has a great temporal inertia. In addition, the regional electricity demand in Brazil is price inelastic. This result reveals that tariff increase policies, such as System of Tariff Flags, will have inexpressive effects on the conservation of energy or potential energy in Brazil

Electricity consumption also showed little sensitivity to variations in income. On the other hand, the elasticity related to the number of units served seems to indicate that programs of access to electricity and legalization of consumption substantially impact the electricity demand. This result is important not only for the Utilities but also for the BES agents who plan

the supply and distribution of electricity, after which they can anticipate the variations in demand due to the increase of units connected to the grid.

The temperature proved to be an important driver to explain the regional demand for electricity, being one of the main concerns of BES agents. In the case of an average variation of 1°C in the temperature there will be an impact of 0.299% on the electricity demand. In the current context in which global climate change has caused extreme temperature levels, this result confirms the importance of including temperature in demand forecasting models. According to INPE forecasts, the temperature tends to increase the electricity demand between 0.90% and 1.49%. This result should be taken into consideration by the Brazilian Utilities.

Finally, it can be concluded that there is a spatial dependence on the regional electricity consumption in Brazil. Therefore, the modelling of this dependency is capable of improving the estimated models results. Spatial spillovers suggest the existence of interaction between neighbouring regions regarding socioeconomic activities, lifestyle and consumer behaviour resulting in similar patterns of regional demand for residential electricity.

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